THERMAL FATIGUE AND INSTRUMENT DESIGN (A)

The die casting shop of Hewlett Packard began to experience premature failures of some dies used in their new cabinet program. Bob Johnson called Professor Fox at Stanford University to ask for his opinion of their suggested remedy for improvement of die life, and to hear his suggestions for improvement.

The names of people in this case are fictitious.

E. I. du Pont de Nemours and Company
The General Electric Foundation
IBM
Olin Corporation Charitable Trust
Union Carbide Corporation

THERMAL FATIGUE AND INSTRUMENT DESIGN (A)

In August 1974, Robert Johnson of Hewlett Packard called Professor George Fox at Stanford University to ask his advice on a problem concerning failure of die casting dies.

Johnson was working on the manufacture of a system for housing electronic instruments, a relatively new cabinet program in which the frames were die cast of aluminum. Exhibit 1 (drawing of a frame); Exhibit 2 (photo of a die casting).

The problem arose when die components, used for casting front frames, failed prematurely, one in particular, after 6500 castings. Exhibit 3 (photo of a broken die component). Johnson suspected that the problem was the result of inadequate temperature control of the die casting dies. According to Johnson, die temperature is probably the least controlled variable in the die casting process and as a result contributes most to the traditional concept that "Die casting is more an art than a science." H-P heated their dies with radiant heaters. Johnson believed that a different die heating system would cut down on thermal stress and improve production. He was considering heating by hot fluid circulating through holes in the dies, and had obtained data for such a heating system and estimated its cost. He now wanted an outside opinion on the expected increase in die life, so that he could compare the cost of the heating system to the value of the expected improvements. He asked Professor Fox to study the problem, suggest solutions, and forecast the effects of suggested remedies.

Fox visited Johnson at Hewlett Packard where he was given more details of the die failure and some background of the cabinet program.

H-P's Cabinet Program

For fifteen years Hewlett Packard had been manufacturing a prize-winning system for housing electronic instruments. The basic cabinet structure in the system was composed of die cast side frames with front and rear sheet metal panels. The top, bottom and side covers were made of vinyl clad aluminum, which can be perforated, when necessary to achieve adequate cooling. It was assembled with spring lock screws in counter-sunk holes. Different divisions of Hewlett-Packard

sometimes used separate die cast front panels.

The system was good for its time, but the need for improvements in packaging more sophisticated instruments became obvious and, in 1969, Hewlett-Packard's Corporate Industrial Design Group, having responsibility for planning, implementing and maintaining those elements which insure a successful company product image, embarked on a preliminary investigation to see what time and dollars would be required to develop a new modular package. With management approval, the design group asked each major division of H-P to send a person for a two year period to work on the team which was to design two new cabinets from which one would be selected after the prototypes were built and tested.

After about 8 months of investigating, the group was able to define the problems, to set priorities, and to establish the design criteria. Approximately 15 man years were spent in developing the new system.

One of the early requirements established by the group for the new system was that it have good thermal capacity. Because the power (watts) input produced a certain temperature increase in a given cabinet size, specifications were set to limit that temperature rise to 10°, 15°, or 20° TC above room temperature. Example: 12-1/4 X Full Mod. (20 watts = 10° rise, 59 watts = 15° rise, 71 watts = 20° rise). It was then left up to Electrical Engineering to choose from these specs the cabinet that best filled the requirement.

Excessive radio frequency waves both from and to cabinet-encased instruments were a major reason for changing from the old system. The instruments must be shielded against waves of frequency up to 1100 MHz. To meet these requirements much of the aesthetics was postponed until the design of the wave entrapments had been fully explored and chosen. It was too expensive to seal the old cabinet.

Tighter tolerances had to be established so that interchangeability of components could be accomplished and instruments could be stacked and locked together.

The System I sheet metal structure was not strong enough to pass Military Specifications which was necessary for the new cabinet.

Two prototype systems were designed by the team. They were built and tested for Radio Frequency Interference (RFI), and for MILSPECS, Class A for shipboard use (400 g, 2 millisecs, 5 to 55 Hz). Both designs showed major improvements

from the System I structure.

The Manufacturing Engineering Group did the costing as their input to the selection decision. They found that inventory costs were the major difference in the cost of the two systems, one having more common usage parts.

The new cabinet was selected on the basis of electric and thermal performance, strength of frame, flexibility of design, and cost. The appearance of the finished cabinet was the last part of the product to be considered, as many of the design features above dictated much of the aesthetics.

Listed below are the characteristics of the old "prize-winning" cabinet (System I), and the new generation cabinet (System II) which was selected as best of the 2 team-designed prototypes.

SYSTEM I	SYSTEM II
1. Die cast side frames 2. Sheet metal front and rear panels 3. Vinyl clad aluminum top, bottom and side covers.	1. Die cast front and rear frames 2. Die cast side struts 3. Vinyl clad aluminum top, bottom and side covers.
Using sheet metal as part of the structure forced reliance on sheet metal covers for the additional strength needed.	Using 2 cast frames and 4 cast struts provides a very rigid structure without covers.
Loose sheet metal tolerances lead to misalignment of other mating parts and to fitting conditions in which parts may be too tight or too loose.	Die cast tolerances make possible the inter-change of components without any rework. The fits are always the same.
Not modular - Sub modules did not justify out to a full module height, width or depth.	Truly modular in every aspect.
Assembly process inflexible. All components must be assembled in sequence. Service good from top and bottom only.	Every aspect of assembly and service improved. Assembly sequence can vary from instrument to instrument. Service can be accomplished from all six sides.
RFI shielding was not critical when designed. Later it proved to be very difficult and expensive to seal the cabinet when required.	RFI shielding was very critical and of prime consideration in designing System II. Basic cabinet is almost as good as System I cabinet sealed. Additional shielding can also be added when needed.
Plug-ins are a dedicated structure when used in the System I cabinet.	All plug-ins are modular and will fit any cabinet. They use the same locking mechanism and guides.

Die cast frames form the front, rear, and sides of the new cabinet which has sheet metal top, bottom and side covers. (See Exhibits 4 and 5). The modular frame components are interchangeable within the cabinet program.

The design group wanted a dull chrome plated finish for the cabinet die cast front frames but chrome plating baths produce effluents which are expensive to dispose of without polluting. As a replacement they are using a two coat aluminum and clear paint system while they are developing an electrostatic powder coating. In this new process, a positive and negative charge are used to spray a powder which clings to the metal. The part is then run through a 420° oven which chemically fuses the powder to the castings. Hewlett Packard finds this an ideal finish for this particular product because the process itself is selective. The charge will not deposit powder into the glitches (RFI grooves), (see Exhibit 1, page 2 of 2 for detail of glitch; Exhibit 3 for photo showing glitch area) which are areas where an insulating finish is not wanted. The effectiveness of the grooves for RFI shielding depends upon the metal to metal contact of the parts.

They use a vinyl coating on the sheet metal top, bottom, and side covers of the cabinet (Exhibit 6). The .014" thick film, not a paint, is an extremely durable finish.

System II was designed with options to fill the needs of almost any user. Handles, ears, or straps may easily be added to any module. Front handles and rack mount ears can be easily mounted on the sides of any cabinet. Strap handles can be mounted on the sides on full modules and on the top for sub-modules.

In late 1972, Manufacturing Engineering was told to go ahead with production. The first few hundred frames were made on a Milwaukeematic N/C milling machine at a cost of over $100 per piece. $3,000 was spent for programming specialists.

By November 1974, HP had begun limited production of 6 sizes of die cast front frames and 8 sizes of die cast rear frames. The dies for this program had cost over $1 million. The production yield was poor -- 70% rejects.

One of the difficulties with the process was that the parts were large, but had thin walls, using a small volume of metal. Rapid temperature change from molten aluminum to thin die cast walls causes cracks and porosity. A small volume of metal injected into the mold allows part of the metal to cool so quickly that the remainder of the metal cannot flow into the mold.

Another difficulty was breakage of dies. This was the problem which prompted Johnson's call to Fox.

Die Life Improvement

After his visit with Bob Johnson, Professor Fox left Hewlett Packard with a list of operation parameters (Exhibit 7), drawings of frames, some die designers' data on thermal stresses, and other information which he would need to answer Johnson's questions and to suggest solutions to the problem.

After thinking about the failures, Professor Fox began to believe that the "glitches" were the main contributors to the problem. His calculations bore out his concern. Glitches are used to produce firm metallic contact between frames and top, bottom, and side covers of the cabinets. This provides RFI shielding. Therefore, the glitches cannot be eliminated. Fox tried to change their design to decrease the thermal stresses.

Professor Fox made an appointment with Johnson to tell him his conclusions. Johnson felt it would be worth the extra effort and cost to have a more formal report (ECL 221B). Fox sent his report to Johnson who distributed it to members of the group (Exhibit 7, memo to design group). When the job was completed, Professor Fox received a HP-65 calculator which he preferred to a money payment.

The following suggestions for improvement made by Johnson and Fox were implemented by Hewlett Packard:

(1) They bought two Mokon die heating units for their Wotan die casting machine. The new heating units heat to 450° in 30-60 minutes. Previously it had taken 3 hours to heat dies to 450°. After reaching the temperature the heaters are turned off.

(2) They maintain a die temperature of 450° F and a surface temperature of approximately 600°F by cycling of the die cast machine without heat transfer fluids.

(3) They changed the alloy from 390 (holding temperature 1400°F) to 360 (holding temperature 1300°F).

(4) They increased the width of the glitches to .050"; they increased radii to .030".

The suggestions were incorporated in one of the dies on a trial basis and were found to be of great help. As dies come in for rework, the modifications are made. HP is now running their modified dies with more success as far as

down time and rejects are concerned. (Rejects are now around 2%-5% rather then 70%.)

In May 1976, HP responded to a request for information on their progress with the glitch problem:

> As far as the glitch problem is concerned, cracking around the glitches is still there and we may have to live with it until a new solution to RFI shielding is found. The solution Professor Fox suggested is in the right direction as it took a number of production runs before the cracks started to propagate again. But, with our design as it is now, we cannot modify the glitches further without losing some of our benefits. We are at this time looking at the possibility of putting the glitch configuration on the sheet metal covers. In doing so, we could eliminate the casting glitches altogether. And we all agree this would be the best solution of all.

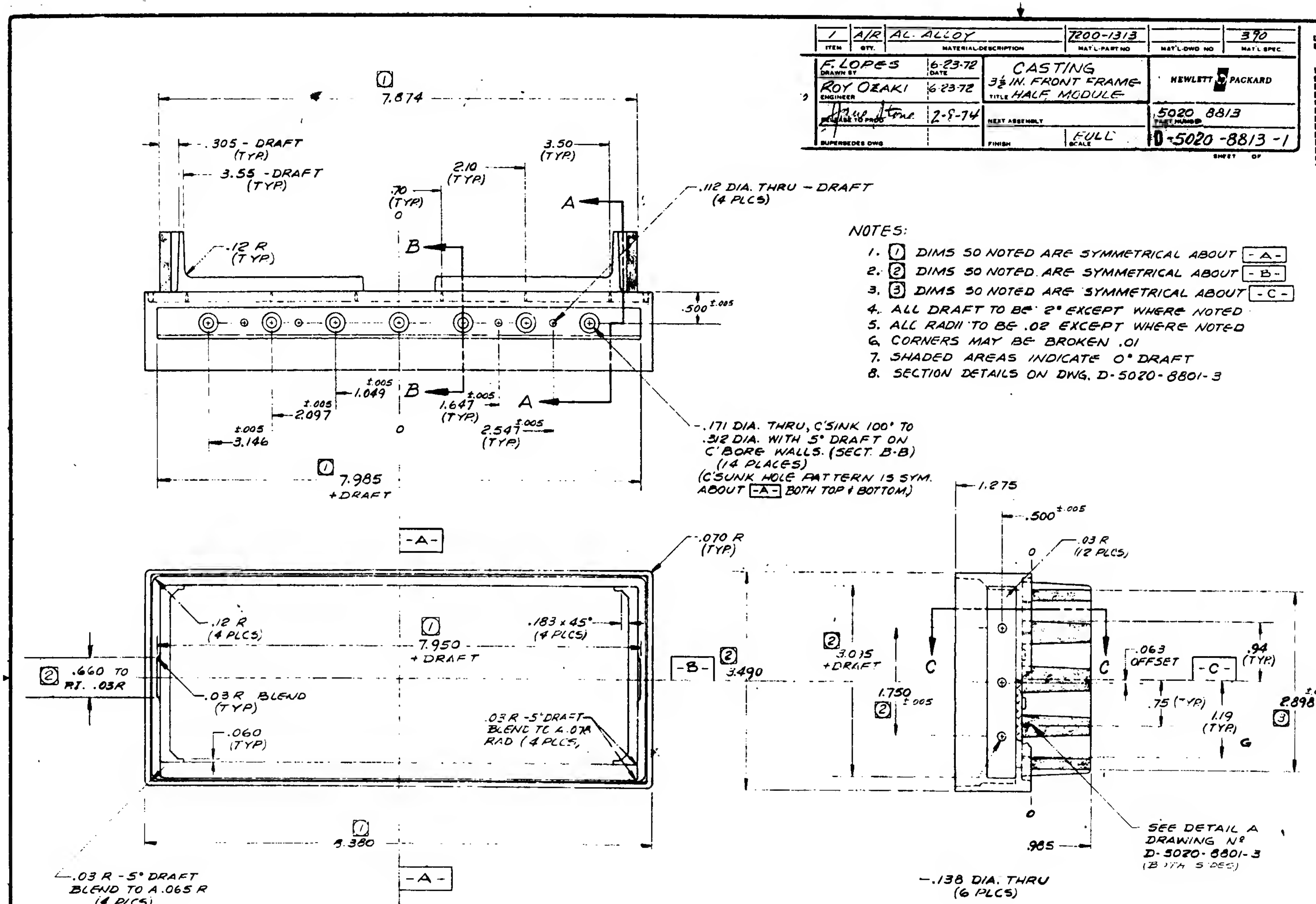

1 A/R AL. ALLOY 7200-1313 390
ITEM QTY. MATERIAL-DESCRIPTION MAT'L-PART NO MAT'L-DWG NO MAT'L SPEC
F. LOPES 6-23-72
ROY OZAKI 6-23-72
2-8-74
CASTING
3½ IN. FRONT FRAME
HALF MODULE
HEWLETT PACKARD
5020 8813
FULL
D-5020-8813-1
NOTES:
1. ① DIMS SO NOTED ARE SYMMETRICAL ABOUT -A-
2. ② DIMS SO NOTED ARE SYMMETRICAL ABOUT -B-
3. ③ DIMS SO NOTED ARE SYMMETRICAL ABOUT -C-
4. ALL DRAFT TO BE 2° EXCEPT WHERE NOTED
5. ALL RADII TO BE .02 EXCEPT WHERE NOTED
6. CORNERS MAY BE BROKEN .01
7. SHADED AREAS INDICATE 0° DRAFT
8. SECTION DETAILS ON DWG. D-5020-8801-3
.112 DIA. THRU – DRAFT (4 PLCS)
.171 DIA. THRU, C'SINK 100° TO .312 DIA. WITH 5° DRAFT ON C'BORE WALLS. (SECT. B-B) (14 PLACES)
(C'SUNK HOLE PATTERN IS SYM. ABOUT -A- BOTH TOP & BOTTOM)
7.874
.305 - DRAFT (TYP)
3.55 - DRAFT (TYP)
.12 R (TYP)
3.50 (TYP)
2.10 (TYP)
.70 (TYP)
.500 ±.005
1.049 ±.005
2.097 ±.005
3.146 ±.005
1.647 ±.005 (TYP)
2.547 ±.005 (TYP)
7.985 + DRAFT
.070 R (TYP)
.12 R (4 PLCS)
7.950 + DRAFT
.183 x 45° (4 PLCS)
.660 TO PT. .03R
.03 R BLEND (TYP)
.060 (TYP)
.03 R - 5° DRAFT BLEND TO A .07R RAD (4 PLCS)
3.380
.03 R - 5° DRAFT BLEND TO A .065 R (4 PLCS)
3.490
3.015 + DRAFT
1.750 ±.005
1.275
.500 ±.005
.03 R (12 PLCS)
.063 OFFSET
.94 (TYP)
.75 (TYP)
1.19 (TYP)
2.898 ±.005
.985
.138 DIA. THRU (6 PLCS)
SEE DETAIL A DRAWING NO D-5020-8801-3 (BOTH SIDES)

EXHIBIT 1

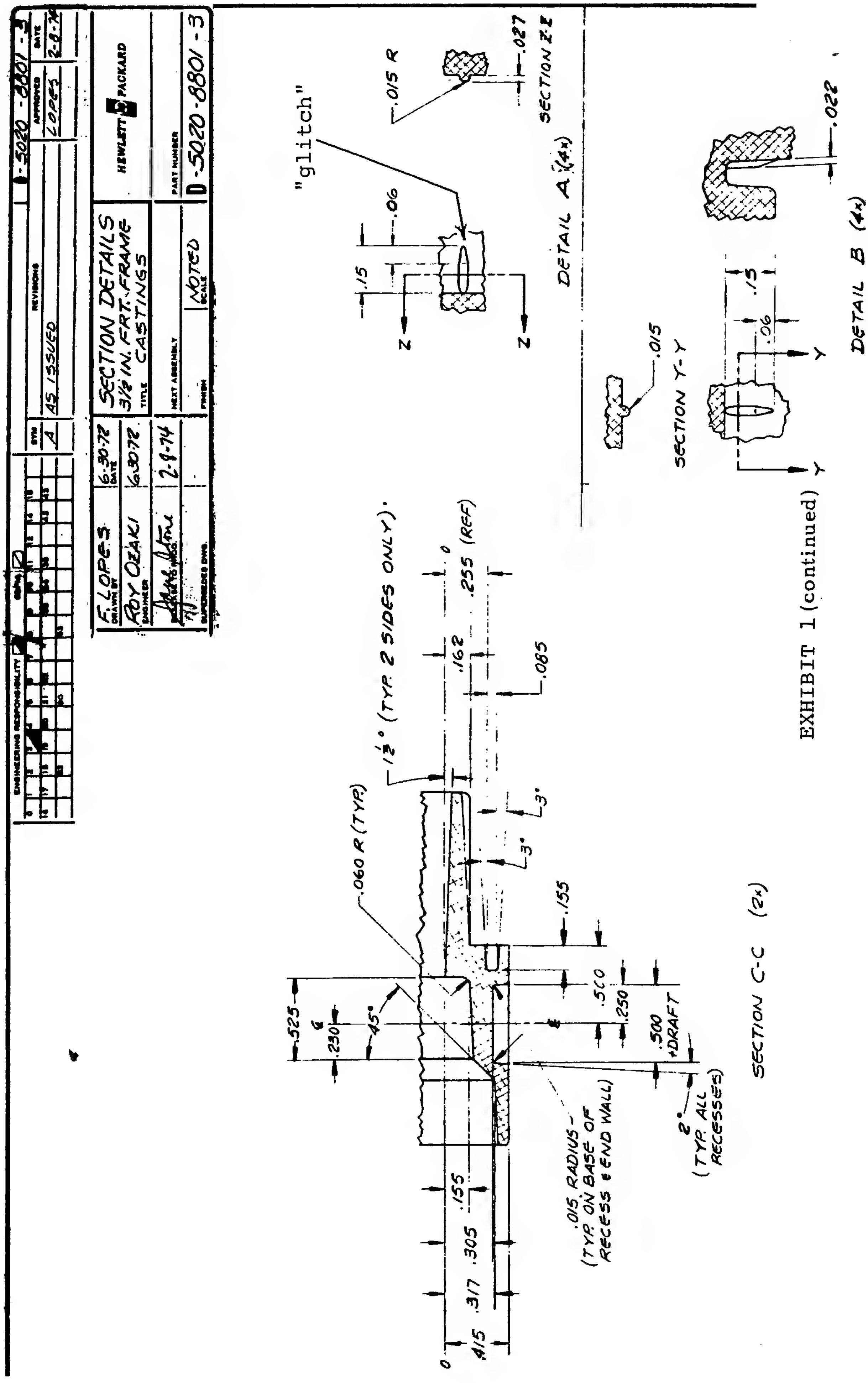


EXHIBIT 1 (continued)

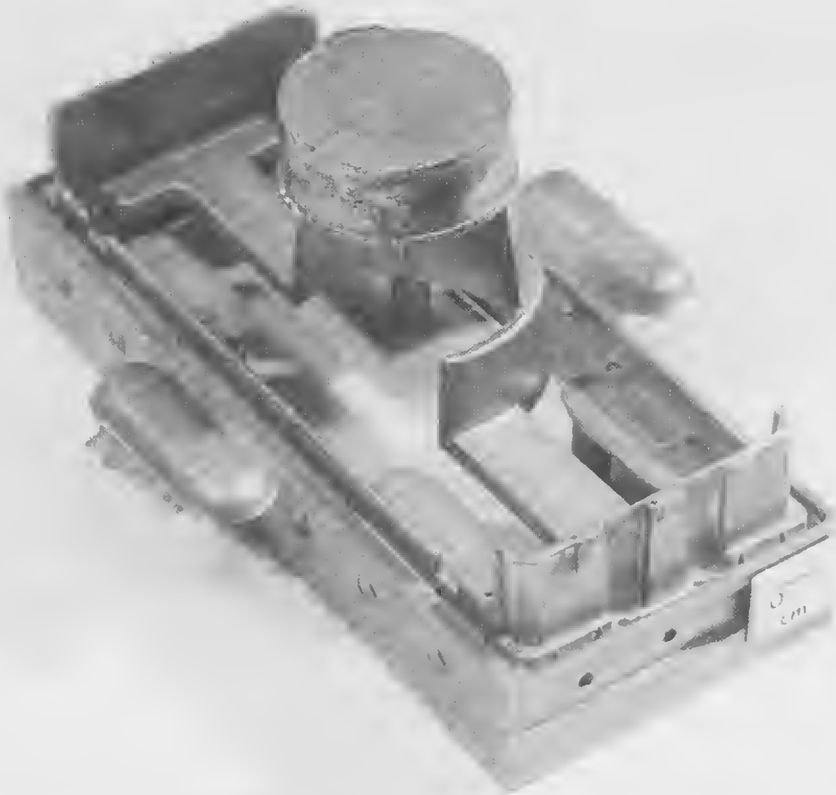


EXHIBIT 2

Front Frame Casting

EXHIBIT 3

Broken Die Component

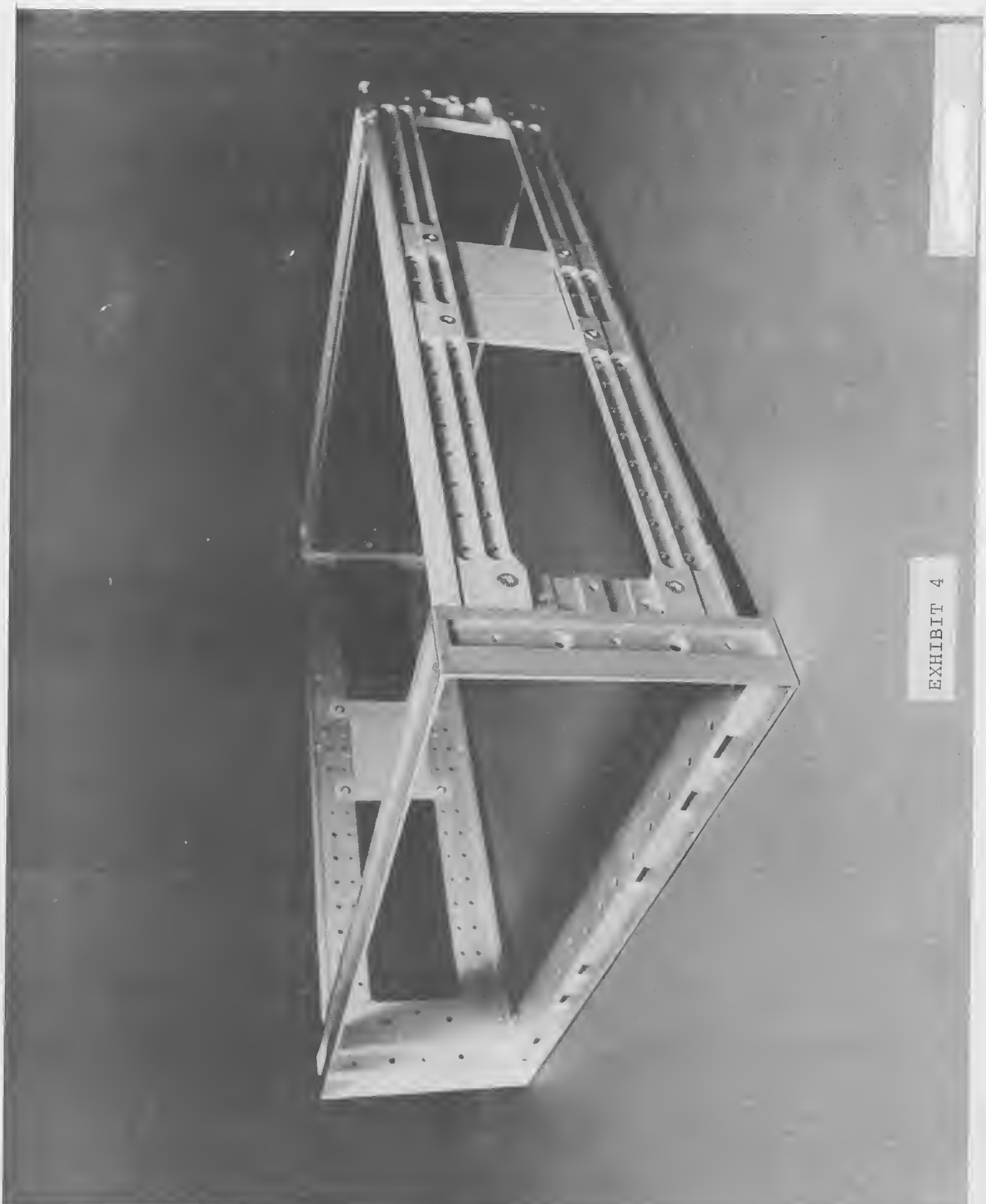


EXHIBIT 4

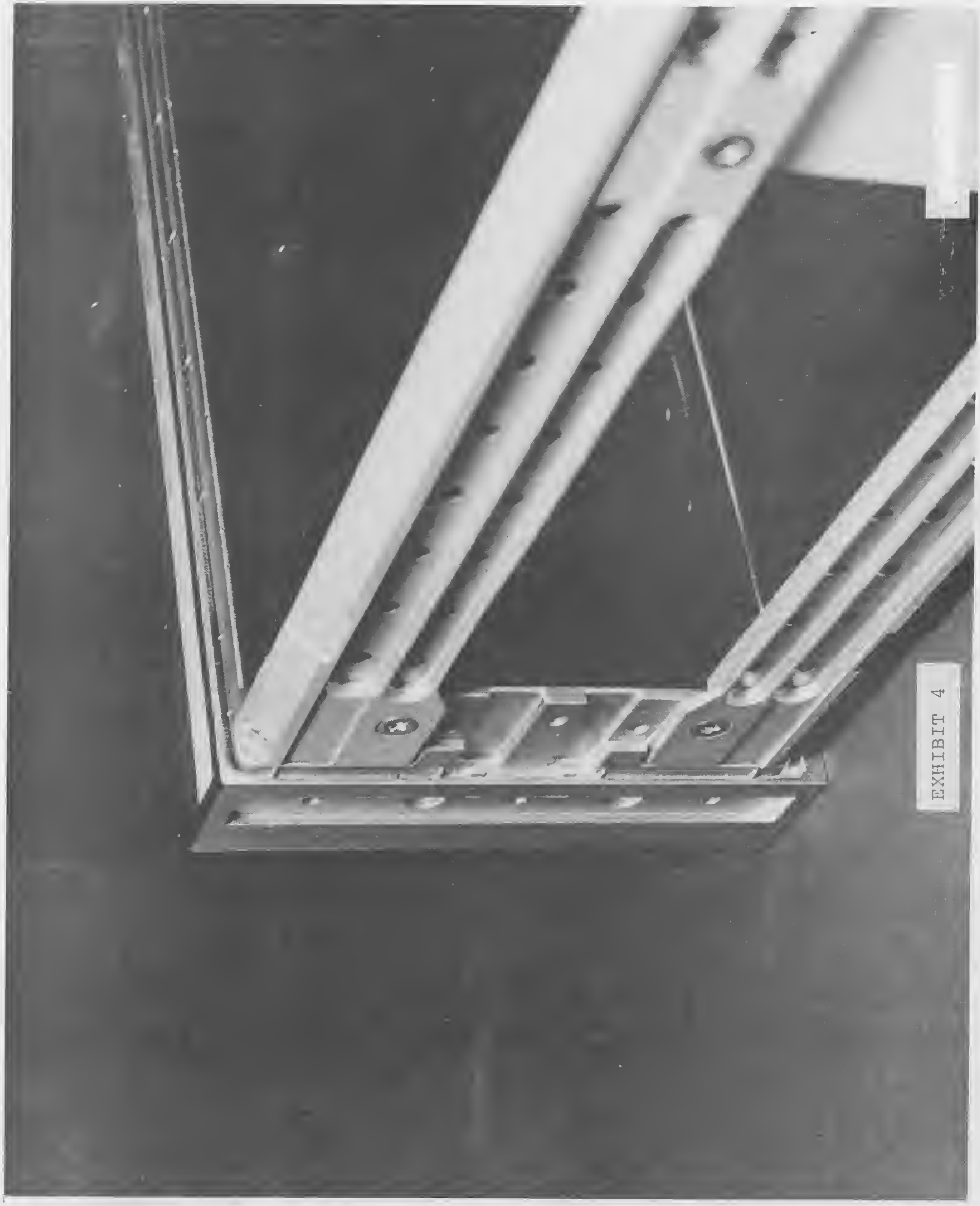
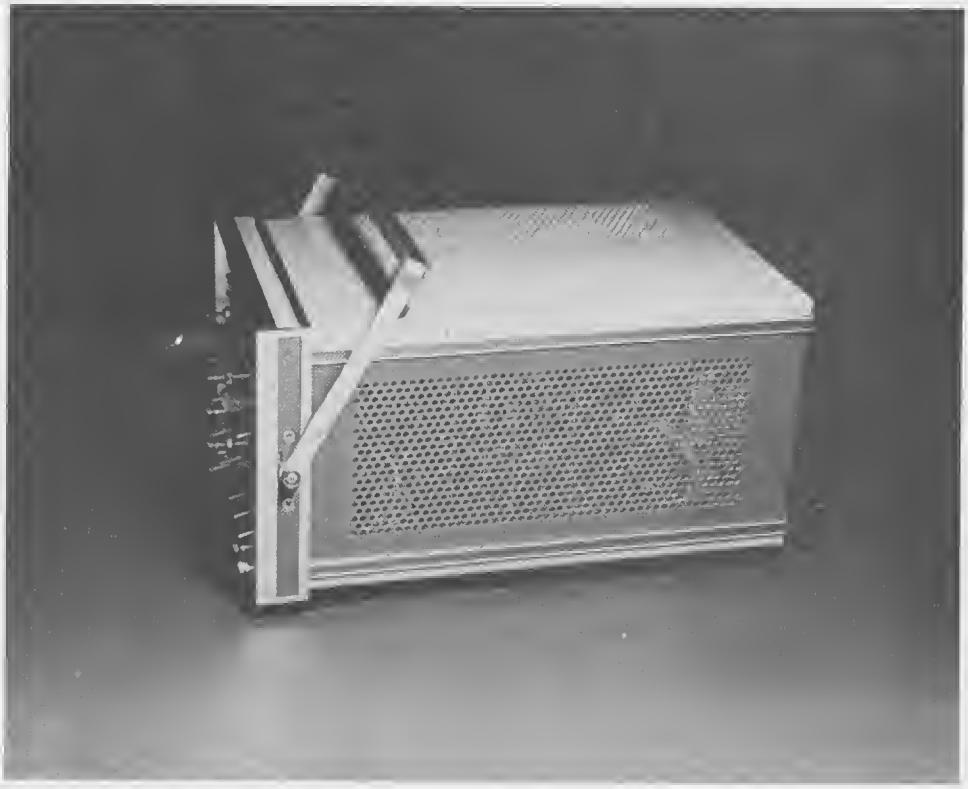


EXHIBIT 4



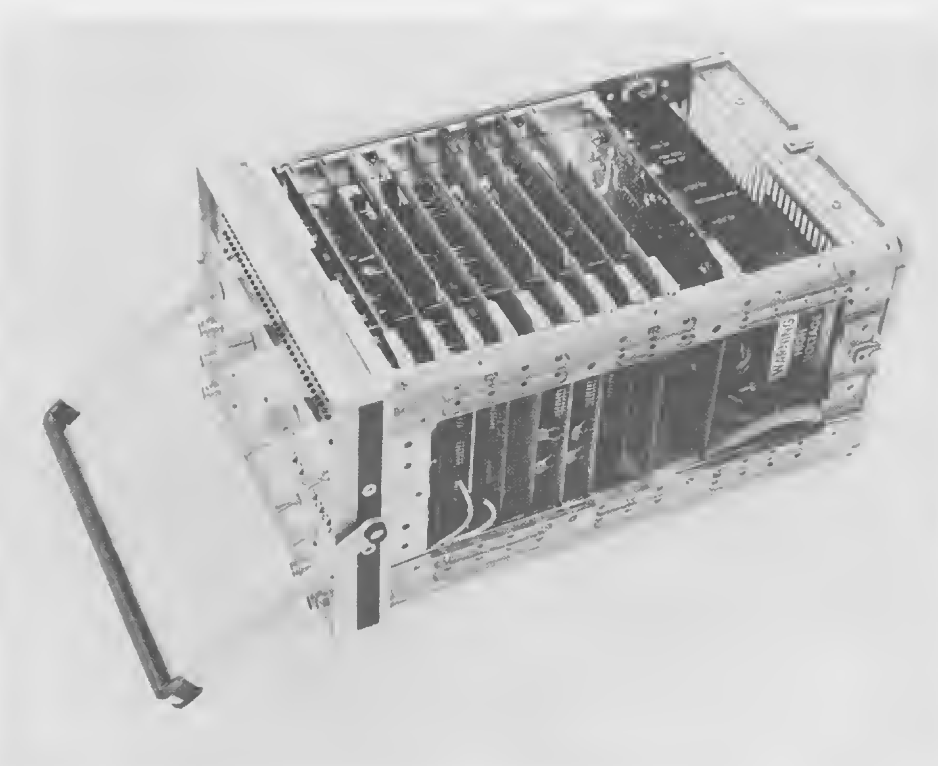


EXHIBIT 5

EXHIBIT 6

OPERATION PARAMETERS

Metal	360 aluminum
Temperature of aluminum entering die	1250 degrees F.
Pressure of aluminum entering die	10,000 psi
Velocity of aluminum entering die	150 ft./sec.
Cycle time (overall)	34 sec.
a. Die open	20 sec.
b. Die closed	
1. Shot preparation	8 sec.
2. Aluminum in die	6 sec.
Die operating time	4 hours (min.)
Die preheat time	2-1/2 hr. (average)
Die operating temperature	(100° - 600°F)
Die operating temperature	(200° - 750°F)

We are looking into the possibility of installing a system which would preheat our dies to approximately 600 degrees Fahrenheit and maintain the average die operating temperature at the desired level.

EXHIBIT 7

(Excerpt from a Hewlett Packard memo)

October 17, 1974

From: Bob Johnson

re: CABINET PROGRAM DIE LIFE REPORT

Attached is a copy of Professor George Fox's report on the premature failure of the 3-1/2 x 1/2 front frame die.

As requested the report provides a basic understanding of the causes of failure, suggestions for improvement and an estimate of the expected life increase produced by such improvements.

In addition, a study of the methods used in determining the requested information provides:

1. An insight into the sensitivity of the many variables

2. An understanding of the effect of die temperature, steel yield strength, corner radii, fluid pressure and change in cross section on die life.

The information contained in this report will be of great value in improving the die life of the cabinet program dies and any other dies if applied correctly.

RJ:js

Attachment

EXHIBIT 8

THERMAL FATIGUE AND INSTRUMENT DESIGN (B)

9 October 1974

Hewlett Packard
395 Page Mill Road
Palo Alto, California 94306

Attention: Mr. Robert Johnson

Dear Mr. Johnson:

With this letter I enclose my report on your die (5208813) which broke prematurely.

According to my figures the most certain remedy would be elimination of the "glitches"; the use of generous radii in the glitches would be the next best remedy.

The round figures on expected life increases which are given in this report are based on many assumptions about the time histories of temperature gradients and pressure differences, about the resulting stress distributions, and about the fatigue properties of the material. I do not want to claim high precision for an analysis based on so many assumptions and would like you to interpret the roundness of my figures in this light.

As the assumptions which I made seem reasonable I expect the quoted figures to be of the right order of magnitude and in about the right proportions to each other.

Sincerely yours,

George Fox
Professor

Report on premature failure of a diecasting die.

G. Fox
8 October 1974

HEWLETT PACKARD DIE

5020-8813

I. THE PROBLEM

This die develops cracks and then breaks in the area of the detail "A" defined on drawings 5020-8813-1 and 5020-8801-3.

II. SUMMARY AND BACKGROUND

The following notes provide a semi-quantitative understanding of the causes of the failure, suggestions for improvements and an estimate of the expected life increase produced by such improvements.

The possible improvements are:

a) preheating to 600° F instead of 300° F.
b) radii increased to 0.020 from 0.010
c) stress relieving grooves as suggested on attached sketch dated 3 October 1974
d) omitting the glitches.

Expected life increases are

from a)	50%
from b)	150%
from c)	50%
from d)	more than 500%

Combinations of these improvements can be expected to produce a life longer than the product of the single factors. For instance:
a and b (or b and c) should produce more than 275% increase (2.75 = 1.5 x 2.5 - 1)

This work was requested by Mr. Robert Johnson by letter dated August 30, 1974 and subsequent conversations. Conclusions have been informally communicated to Mr. Johnson on 12 September and on 4 October 1974: they are modified in details in these notes.

III. Relevant Data

The life of the die was reported to be approximately 6500 castings before it broke. The die was reported to operate in runs of about 2 hours, with 120 parts per hour and with an operating temperature of 600° or 700° F reached after about 50 shots.

Drawings 5020-8813-1 to 3 define the part made by the die.

A part of the broken die was examined with low power magnification.

The aluminum was reported to enter the die at 1250° F and 15,000 psi.

The die material was reported to be Carpenter No. 883, with the following properties listed by Carpenter:

Coefficient of thermal expansion:
7.8 parts per million per deg F (at 500 to 1200° F)

Modulus of elasticity:
27 million psi (at 900° and 600° F)

Yield strength at 50 Rockwell C at room temperature:
210,000 psi

Ratio of hot and cold tensile strength at 1000/70° F is 180/270 psi or 67%

Ratio of hot to cold Brinell hardness at 1000/70° F is 375/560 or 67%

Fatigue properties for this material were not available. A probable strain-life curve was constructed from available data. (See Appendix A)

IV. Analysis:

The thin ridge of the die, surrounded by metal on both sides, is assumed to reach 1000° F. At this temperature the yield strength is expected to be 67% of the room temperature yield strength of 140,000 psi.

Yield strain will be 140/27,000 = .52%

Thermal strain* is estimated, assuming an effective temperature difference of 500° F between the ridge and neighboring parts of the die. If the glitches did not exist, this would produce a thermal stress of 500 x 7.8 x 27 = 105,300 psi or a strain of 500 x 7.8 parts per million = 0.39%.

The glitches will increase this in two ways:

a) for equilibirum, the average stress in the smaller cross section must be higher than the average stress in the larger cross section.

b) the transition from one to the other cross section produces strain concentrations in the corners.

For the proportions of the ridge with the glitch, the strain increase caused by a) is 54% at the assumed temperature, as estimated in appendix B

The strain increase caused by b) is estimated to be 50% for the nominal radii of 0.015, 70% for radii of 0.010 which seem to exist in the die, and 35% for the radii of 0.020 which are the upper limit according to the drawing.

The maximum thermal strain in the corner of the glitch groove, caused by 500° F effective temperature difference, is then

.39% x 1.54 x 1.7 = 1%

From appendix A we would expect a life of 20,000 cycles at this strain range.

At the beginning of each run, the temperature differences may be twice as high and the strains more than twice as high. The total production of 6500 pieces corresponds to 27 two-hour runs.

If the high temperature differences persist for 10 pieces at the beginning of each run, we would have 270 cycles of this high strain. Assuming the strain range to be 2.5%, we would expect a life of 630 cycles of this magnitude.

---

* "Strain" here means only that part of the total strain which corresponds to the thermal stresses; it does not include free expansion or contraction of the whole die.

In addition to these thermal strains, we shall estimate the strains imposed by an unbalanced fluid pressure.

The maximum pressure is 15,000 psi. The pressure difference between the two sides of the wall is assumed to be 7.500 psi for some time during each molding operation.

In the corner of the glitch groove this would produce a strain of 0.2% according to appendix C.

The timing of the two strain peaks, thermal and mechanical, is uncertain. Their effects would add to a total strain range of 1.2% if either a tensile mechanical strain occurs near the time of low thermal compressive strain or, if the mechanical strain were compressive, by a reflected pressure wave, at the time of high thermal strain. Assuming that this happens, that our assumptions of temperature differences and pressure differences are correct, and that the estimated strain-amplitude versus life to failure curve is correct, the total strain range of 1.2% (amplitude 0.6%) would produce failure in about 5000 cycles and the strain range of 2.7% (for the cold die) would produce a life of about 500 cycles. Assuming the damage done by one cycle to be the inverse of the number of such cycles which would produce failure, the damage per 2 hour run would be:

$$10/500 + 230/5000 = 0.02 + 0.046 = 0.066$$

The number of runs to failure would be

$$1/0.066 = 15$$

And the number of parts made at failure

$$15 \times 240 = 3600$$

My estimates of strains are probably somewhat too high.

V. Estimates of Effects of Changes

a) preheating to 600° F

This would eliminate the few cycles of high thermal stress which were estimated to produce 0.02/0.066 = 30% of the damage. Life would then be increased in the ratio $1/(1 - .3) = 1.43$ or approximately 50% increase in life.

b) increasing the actual glitch radius from 0.010 to 0.020 would decrease the strains in the ratio of 1.35/1.70 and increase the life in the ratio $(1.7/1.35)^4 = 2.5$ times.

The estimate of 2.5 times was based on the approximate slope of the strain-life curve in the region between 500 and 5000 cycles. Another estimate relates a strain range of 1.2% to the reported life of approximately 6000 cycles; it then reduces this strain range to 1.2% x 1.35/1.7 = 0.95% and finds a life of 22,000 cycles for this strain range, corresponding to the larger radius. This would be a 250% life increase. The more conservative value of 150% is reported in the summary.

c) Strain relief grooves as in attached sketch would decrease the thermal strains for elastic conditions in the ratio of 1.14/1.37 = 0.83.

Total strain ranges would be decreased from
2.5% + 0.2% and 1% + 0.2% to
2.2% + 0.2% and 0.9% + 0.2%

and the damages decreased in the ratios

$$(2.4/2.7)^2 \text{ and } (1.1/1.2)^4$$

to become

$$0.79 \times 0.02 + 0.7 \times 0.046 = 0.05$$

for a calculated life of 20 runs instead of 15, and an increase of 33%

This figure is based on elastic thermal strains without stress concentrations. With plastic strains, which we have, and with the decrease of stress concentration in the corners which is produced by the relief grooves, the life increase will be higher. In the summary it is reported as 50%.

d) Omitting the glitches would reduce the thermal strains in the ratio

$$1/(1.54 \times 1.7) = 0.38$$

and the mechanical strains in the ratio

$$1/1.7 = 0.59$$

The total strain range at the beginning of each run would then be

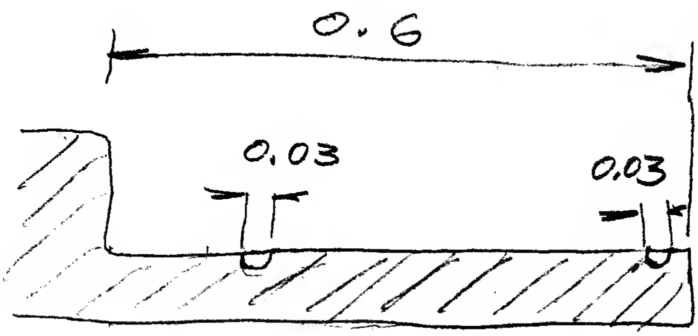
$$2.5\% \times 0.38 + 0.2\% \times 0.59 = 1.1\%$$

This strain range corresponds to a life of 10,000 cycles or 1000 short runs of 10 shots, as compared with the calculated life of 500 cycles or 50 short runs with the present conditions.

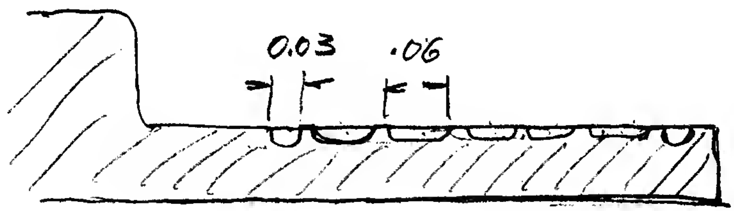
The strains with the warmed-up die would be only 0.38% x 0.12% = 0.5% corresponding to a life of more than a million cycles.

The expected life would be 1000 two-hour runs or 240,000 pieces, or about 70 times the life calculated for the present conditions.

As this result is very high, it should be checked very roughly as follows: Assuming an overall strain decrease to 60% and a slope 1:4 of the log strain-log life curve, we would expect a life increase of $1.67^4 = 7.7$ times. This is far less than the 100 fold increase calculated before but still more than the required life increase to 4 or 5 times the present life.



PRESENT



MODIFIED

grooves tapered to disappear at root

Present $f = 0.06/0.6 = 0.1$

Modified $f = (0.06+0.30)/0.6 = 0.6$

Elastic thermal strain increase factor

$= 1/(g+f-fg)$ with $g = 0.7$

for present shape $1/(.7+.1-.07) = 1/.73 = 1.37$

for modified shape $1/(.7+.6-.42) = 1/.88 = 1.14$

SEE APPENDIX (pages 29 to 31) FOR DERIVATIONS

Estimation of a strain-life curve for Carpenter 883 at 1000 deg F. using the method of SAE Fatigue Design Handbook p 21 to 25

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

$b = -0.085 \ (= -\frac{1}{12})$ $\qquad c = -0.6 \ (= -\frac{1}{1.7})$

$\sigma_f' = \sigma_f = f S_u / (1 - RA)$

$S_u =$ 180 ksi from Carpenter p 131

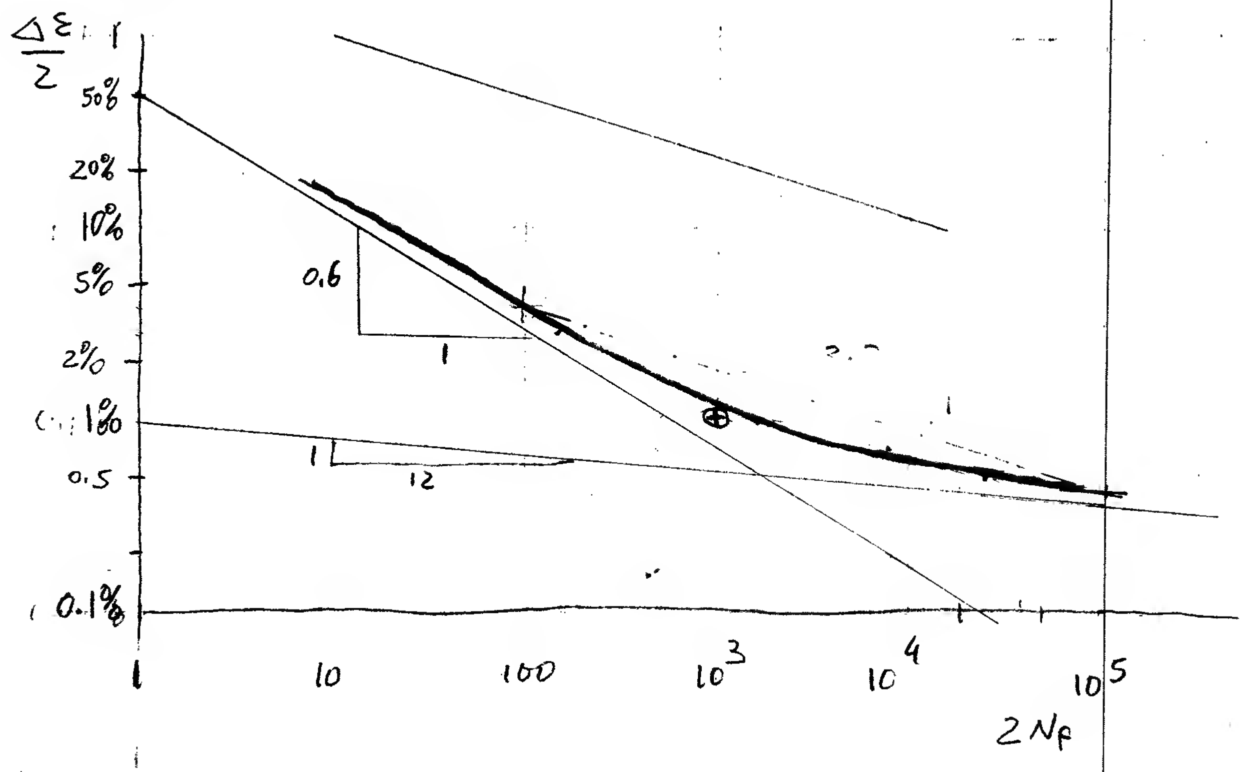
$RA =$ 40% from Carpenter p 130, considering last figure on p 131 on decrease of hardness by heat.

$f = 0.9$ from Dieter, Mech. Metallurgy, fig 9-7

$\sigma_f' = 0.9 \times 180/.6 = 270$ ksi

$\sigma_f'/E = 270/27000 = 1\%$

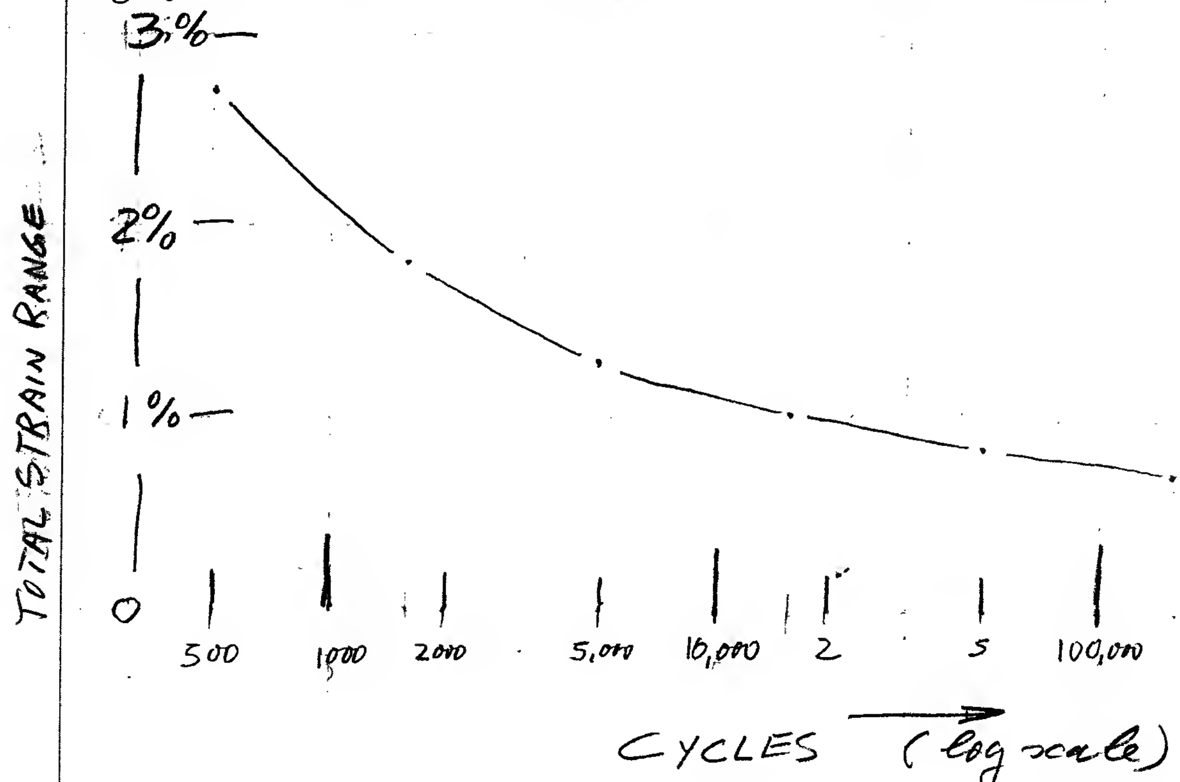
$\varepsilon_f' \approx \varepsilon_f = \ln 1/(1-RA) = \ln 1.67 = 0.51$

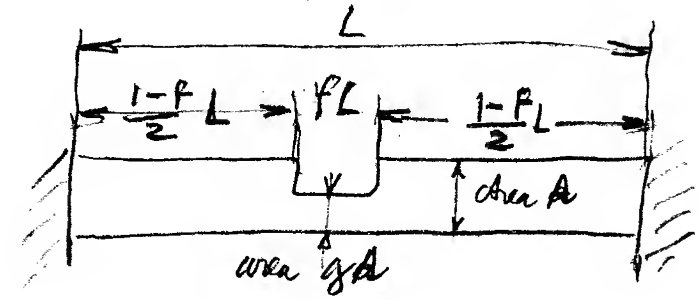


$\frac{2.7}{2.5} = 1.08 \quad \log 1.08 = 0.0334 \quad \log 1.08^{3.2} =$

The lower end of this curve is replotted to larger scales on page 2

$\log 2N_f$	3	3.5	4	4.5	5	5.5
$.085 \log 2N_f$	.255	.297	.34	.382	.425	.467
$2N_f^{0.085}$	1.8	1.98	2.19	2.41	2.66	2.93
$\frac{\Delta\varepsilon_e}{2}$ %	.555	.505	.457	.415	.376	.341
$.6 \log 2N_f$	1.8	2.1	2.4	2.7	3	3.3
$2N_f^{.6}$	63.1	126	251	501	1000	1995
$\frac{\Delta\varepsilon_p}{2}$ %	.81	.4	.2	.1	.05	.026
$\frac{\Delta\varepsilon}{2}$ %	1.365	.905	.657	.515	.426	.367
"cycles" N_f	500	1,580	5,000	15,800	50,000	158,000
log "cycles"	2.699	3.199	3.699	4.199	4.699	5.199





Total compression: $\alpha L = \beta(1-f)L + \gamma f L$

| α = total strain | β = strain in ridge | γ = strain in glitch |

1. Elastic : $\beta \times A = \gamma \times g A$

$$\gamma = \frac{\beta}{g}$$

$$\alpha L = \beta(1-f)L + \frac{\beta}{g} f L$$

$$\alpha = \beta\left[1 - f + \frac{f}{g}\right] = \frac{\beta}{g}\left[g - fg + f\right]$$

$$\beta = g\alpha/(g+f-fg)$$

$$\gamma = \frac{\beta}{g} = \alpha/(g+f-fg)$$

for our die: $f = .06/.60 = .1$ for two glitches

$g = .053/.075 = 0.7$

$$g + f - fg = .8 - .07 = .73$$

$$\gamma = \alpha/.73 = 1.37\alpha$$

The average stress in the cross-section near the glitch then is

$$105{,}300 \times 1.37 = 144{,}000 \text{ psi}$$

or very little above the yield strength.

To take care of conditions beyond yield I will use the secant moduli E' and iterate until the calculated strains correspond to the assumed ratio of secant moduli.

to allow for strains beyond yield strain:

Total strain $\alpha = \beta(1-f) + \gamma f$ as before

but for equilibrium $\beta E_1' A = \gamma g E_2' A$

$$\gamma = \frac{\beta}{g} \cdot \frac{E_1'}{E_2'} = \beta \frac{m}{g}$$

set $E_1'/E_2' = m$

recall: f = ratio ~~ratio~~ of length with reduced thickness to total length

g = ratio of reduced area to full area

m = ratio of secant modulus of low strain part to secant modulus of high strain part

then $\gamma = \frac{m}{g}\beta \qquad \beta = \frac{g}{m}\gamma$

$$\alpha = \frac{g}{m}\gamma(1-f) + \gamma f = \gamma\left[\frac{g}{m} - \frac{gf}{m} + f\right]$$

$$\gamma = \alpha \Big/ \left(\frac{g}{m} + f - \frac{gf}{m}\right)$$

Assume material to be perfectly plastic above yield. (no strain hardening) THIS WILL EXAGGERATE the condition.

Then, for our conditions $E_1' = 27$ million psi

$E_2' = \frac{140{,}000 \text{ psi}}{\gamma} \qquad m = \gamma \times \frac{27{,}000}{140} = 193\gamma$

$$\gamma = \alpha \Big/ \left(\frac{g}{193\gamma} + f - \frac{fg}{193\gamma}\right)$$

$$\frac{g}{193} + f\gamma - \frac{fg}{193} = \alpha \quad - \quad \gamma = \left(\alpha + \frac{fg}{193} - \frac{g}{193}\right)/f$$

The solution depends on the magnitude of α, which in our case is $500 \times 7.8/1{,}000{,}000$

$\alpha = 3.9/1000$

$$\gamma = \left(\frac{3.9}{1000} + \frac{.7}{19300} - \frac{7}{1930}\right) \times 10$$

$$= \frac{.39}{1000} + \frac{7}{1930} - \frac{70}{1930} = \frac{39}{1000} + \frac{3.6}{1000} - \frac{36}{1000}$$

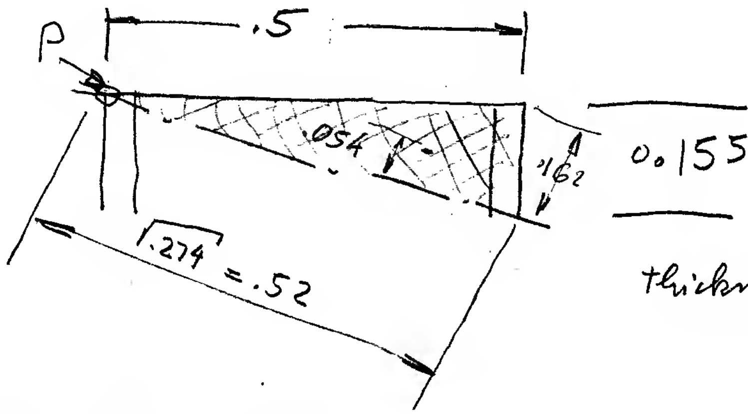
$$= 6.6/1000 = 0.66\ \%$$

with elastic conditions we had $\gamma = 1.37\alpha$

$\gamma_E = 1.37 \times .39\% = 0.534\ \%$

Lets assume $\gamma = 0.6\%$

The strain increase factor is $0.6/0.39 = 1.54$



thickness $\approx \frac{8}{100}$"

Bending stress at point P = $\sigma = 1.7S$

S = nominal bending stress = M/Z

$$M = 7500 \text{ psi} \times \frac{0.52 \times 0.162}{2} \times 0.054 = 17.1 \text{ inlbs}$$

$$Z = \frac{.52 \times 8^2}{6 \times 100^2} = \frac{5.55}{10{,}000}$$

$$S = 171{,}000/5.55 = 31{,}000 \text{ psi}$$

$$\sigma = 1.7 \times 31{,}000 = 52{,}000 \text{ psi}$$

$$\varepsilon_B = 52/27{,}000 = 0.19\% \quad \text{say } 0.2\%$$

1.7 is stress concentration factor for 0.10 radius in the glitch groove.